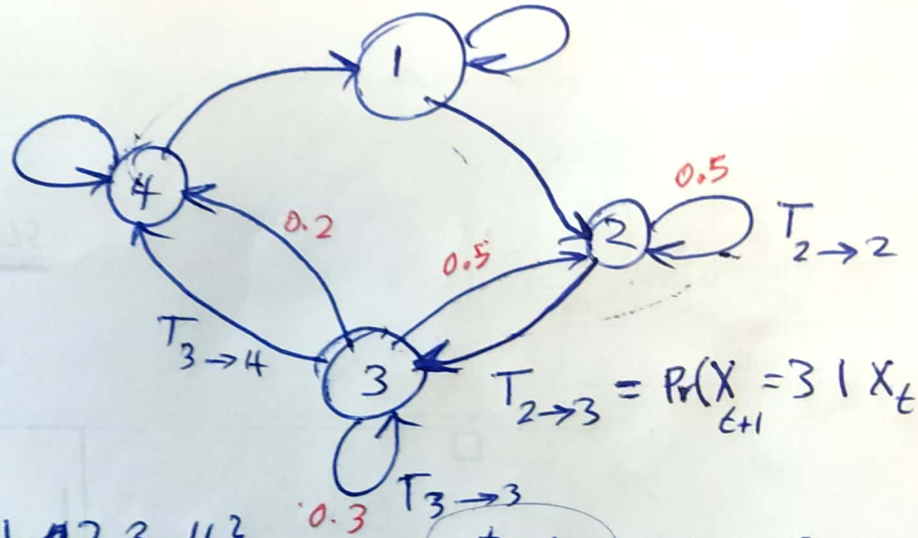
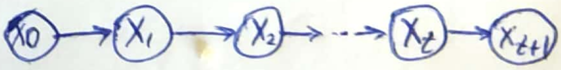


$T_{2\to 3} = Pr(X_{t+1} = 3 \mid X_t = 2)$

$X \in \{1, 2, 3, 4\}$

$p^t(X)$: prob. of being in state X at time t

$$p^t(X) = \sum$$



$$p^t(a) = \sum_b Pr(X_{t+1} = a \mid X_t = b)\, p^{t-1}(b)$$

$$p^t(a) = \sum_b T_{b\to a}\, p^{t-1}(b) \qquad a = 1, 2, 3, 4$$

$$\lim_{t\to\infty} p^t(a) = p^{\infty}(a) = \pi(a) = \pi_a$$

if converge:

$$\lim_{t\to\infty} p^t(a) = \lim_{t\to\infty} \sum_b T_{b\to a}\, p^{t-1}(b)$$

$$\boxed{\pi(a) = \sum_b T_{b\to a}\, \pi(b)} \qquad a = 1, 2, 3, 4$$

$$\pi_a = \sum_b T_{b\to a}\,\pi_b \qquad a=1,2,3,4$$

$$\pi_1 = \sum_b T_{b\to 1}\,\pi_b$$

$$\text{I}\quad \pi_1 = T_{1\to1}\,\pi_1 + T_{2\to1}\,\pi_2 + T_{3\to1}\,\pi_3 + T_{4\to1}\,\pi_4$$

$$\text{II}\quad \pi_2 = T_{1\to2}\,\pi_1 + T_{2\to2}\,\pi_2 + T_{3\to2}\,\pi_3 + T_{4\to2}\,\pi_4$$

$$\text{III}\quad \pi_3 = T_{1\to3}\,\pi_1 + T_{2\to3}\,\pi_2 + T_{3\to3}\,\pi_3 + T_{4\to3}\,\pi_4$$

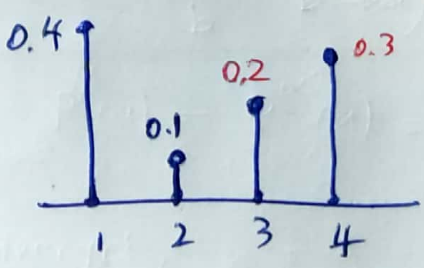
$$\text{IV}\quad \pi_4 = T_{1\to4}\,\pi_1 + T_{2\to4}\,\pi_2 + T_{3\to4}\,\pi_3 + T_{4\to4}\,\pi_4$$

4 equation 4 unknows $(\pi_1, \pi_2, \pi_3, \pi_4)$

3 independent equations

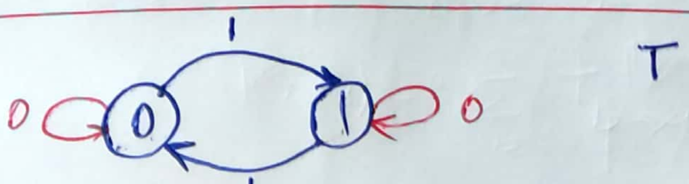
$$\text{V}\quad \boxed{\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1}$$

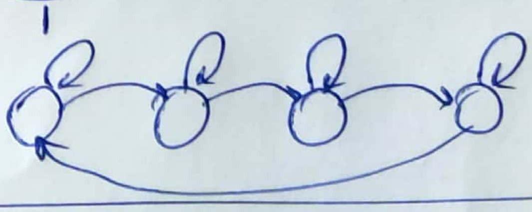
Assume that $\pi_1 = 0.4$ $\pi_2 = 0.1$ $\pi_3 = 0.2$ $\pi_4 = 0.3$



$P(x)$

Stationary Distribution





Markov Chain Monte Carlo (MCMC)

Gibbs Sampling

$P(A, B, C, D, E)$

$$a^{t+1} \sim P(A \mid B=b^t, C=c^t, D=d^t, E=e^t)$$
$$b^{t+1} \sim P(B \mid A=a^{t+1}, C=c^t, D=d^t, E=e^t)$$
$$c^{t+1} \sim P(C \mid A=a^{t+1}, B=b^{t+1}, D=d^t, E=e^t)$$
$$d^{t+1} \sim P(D \mid A=a^{t+1}, B=b^{t+1}, C=c^{t+1}, E=e^t)$$
$$e^{t+1} \sim P(E \mid A=a^{t+1}, B=b^{t+1}, C=c^{t+1}, D=d^{t+1})$$

$$(a^t, b^t, c^t, d^t, e^t) \longrightarrow (a^{t+1}, b^{t+1}, c^{t+1}, d^{t+1}, e^{t+1})$$

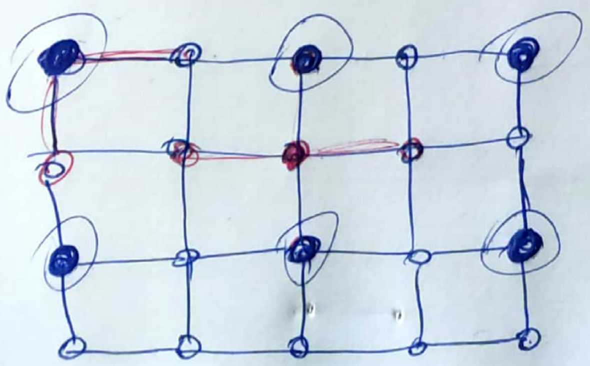
$$P(A \mid B, C, D, E) = \frac{P(A, B, C, D, E)}{\sum_A P(A, B, C, D, E)}$$

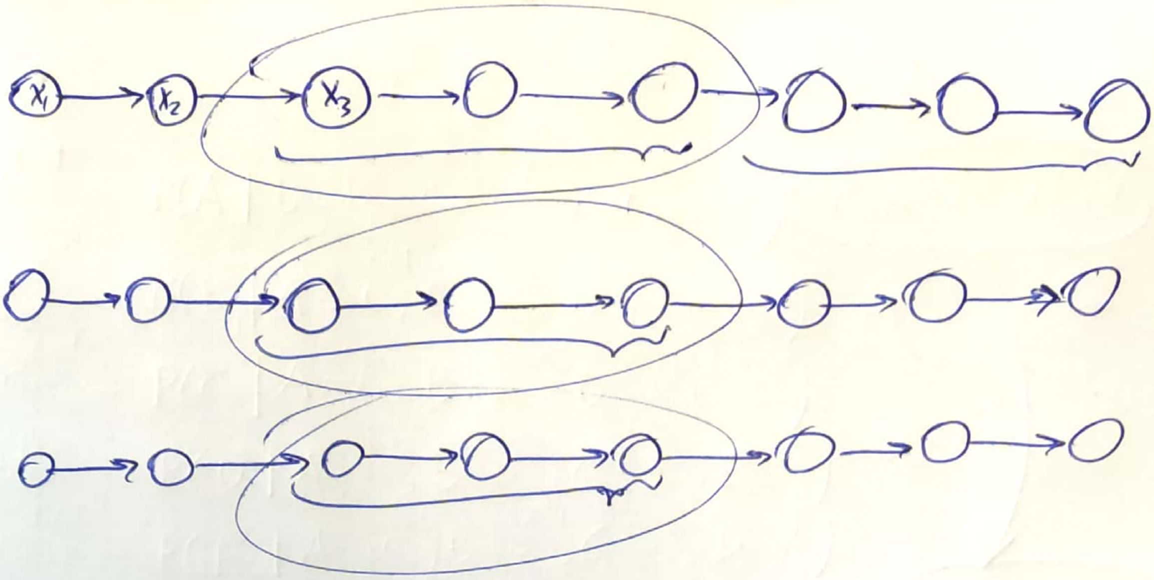
$$P(A, B, C, D, E) = \frac{1}{Z} \Phi_1(A)\, \Phi_2(A, B)\, \Phi_3(B, C)\, \Phi_4(C, D, E)$$

$$P(A \mid b, c, d, e) = \frac{\cancel{\frac{1}{Z}} \Phi_1(A)\, \Phi_2(A, b)\, \Phi_3(b, c)\, \Phi_4(c, d, e)}{\sum_a \cancel{\frac{1}{Z}} \Phi_1(a)\, \Phi_2(a, b)\, \Phi_3(b, c)\, \Phi_4(c, d, e)}$$

$$P(A \mid b, c, d, e) = \frac{\cancel{\frac{1}{Z}} \Phi_1(A)\, \Phi_2(A, b)\, \cancel{\Phi_3(b, c)}\, \cancel{\Phi_4(c, d, e)}}{\sum_d \cancel{\frac{1}{Z}} \Phi_1(a)\, \Phi_2(a, b)\, \cancel{\Phi_3(b, c)}\, \cancel{\Phi_4(c, d, e)}}$$

$$= \frac{\Phi_1(A)\, \Phi_2(A, b)}{\sum_a \Phi_1(a)\, \Phi_2(a, b)}$$





X_1
X_2
X_3